ME 141

Engineering Mechanics

Portion 6

Distributed Force

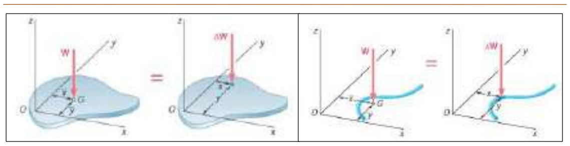
Partha Kumar Das
Lecturer
Department of Mechanical Engineering, BUET
http://parthakdas.buet.ac.bd

The point at which the resultant (total weight of a body) of a set of distributed forces (individual weight of each particle) acts is known as Center of Gravity of the body.

Center of Gravity

- 2D
  - Area
    - Non Homogenous, Non Uniform Thickness
    - Homogenous, Uniform Thickness (Centroid of Area)
  - Line
    - Non Homogenous, Non Uniform Cross-section
    - Homogenous, Uniform Cross-section (Centroid of Line)
- 3D

Center of Gravity (Areas and Lines)



$$\Sigma F_z: \quad W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n$$

$$\Sigma M_y: \quad \bar{x}W = x_1\Delta W_1 + x_2\Delta W_2 + \cdots + x_n\Delta W_n$$

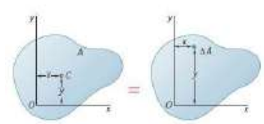
$$\Sigma M_x: \quad \bar{y}W = y_1\Delta W_1 + y_2\Delta W_2 + \cdots + y_n\Delta W_n$$

$$\Sigma M_y: \quad \bar{x}\,W = \Sigma x\,\Delta W$$

$$\Sigma M_x: \quad \bar{y}\,W = \Sigma y\,\Delta W$$

$$W = \int dW \qquad \bar{x}W = \int x\,dW \qquad \bar{y}W = \int y\,dW$$

Centroids of Areas and Lines



Considering a flat homogenous plate with uniform thickness,
$W = \gamma\, t\, A$

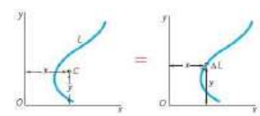
For individual particle,
$\Delta W = \gamma\, t\, \Delta A$

Substituting in the previous equations,

$$\Sigma M_y: \quad \bar{x}A = x_1\Delta A_1 + x_2\Delta A_2 + \cdots + x_n\Delta A_n$$

$$\Sigma M_x: \quad \bar{y}A = y_1\Delta A_1 + y_2\Delta A_2 + \cdots + y_n\Delta A_n$$

$$\bar{x}A = \int x\,dA \qquad \bar{y}A = \int y\,dA$$



Considering a homogenous wire with uniform cross section,
$W = \gamma\, a\, L$

For individual particle,
$\Delta W = \gamma\, a\, \Delta L$

Substituting in the previous equations,

$$\bar{x}L = \int x\,dL \qquad \bar{y}L = \int y\,dL$$

First Moments of Areas and Lines

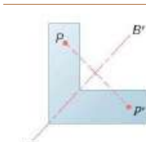
First Moment of Area with respect to x- axis is represented as Q_x, which is the following integral
$Q_x = \int y\, dA = \bar{y}\, A$

First Moment of Area with respect to y- axis is represented as Q_y, which is the following integral
$Q_y = \int x\, dA = \bar{x}\, A$

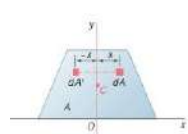
Similar cases hold for lines or wires.

Note:
- If the centroid of an area or line lies on any co-ordinate axis, then the first moment of area or line with respect to that co-ordinate axis is zero.
- If the first moment of area or line with respect a co-ordinate axis is zero, then the centroid of the area or line lies on that co-ordinate axis.
- So, if any axis (regardless co-ordinate axes) can be selected with respect to which the first moment of area or line is zero, then the centroid lies on that axis.

First Moments of Areas and Lines

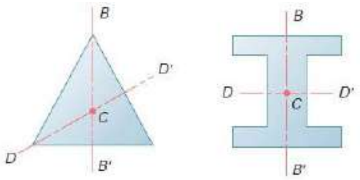


Axis of Symmetry: If any point *P* has its mirror point *P'* with respect to an axis *BB'*, that *PP'* is perpendicular to *BB'* and *BB'* bisects *PP'*, then *BB'* is known as the Axis of Symmetry.



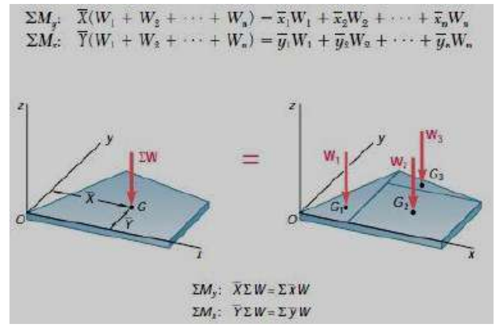
- **The first Moment of Area or Line with respect to a Axis of symmetry is zero.**
- **Centroid lies on the axis of symmetry.**

First Moments of Areas and Lines

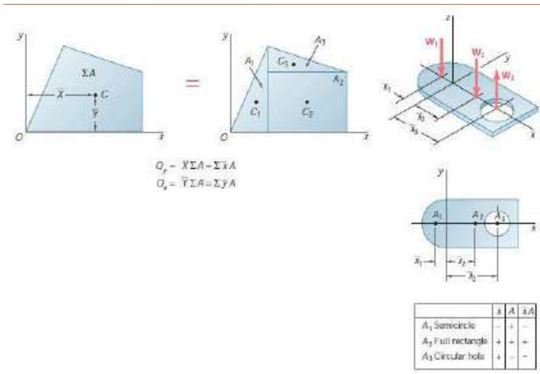


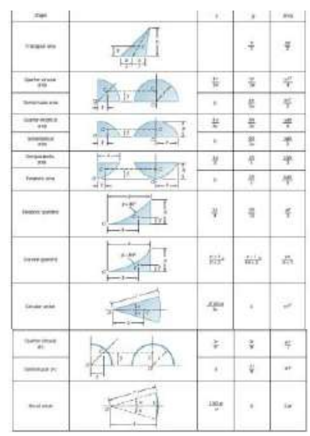
✓ If there are two axes of symmetry, centroid must lye on the intersection point of the axes.

Composite Plates and Wires



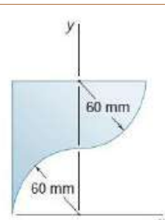
Composite Plates and Wires





Problem 6.1 (Beer Johnston_10th edition_P5.8)

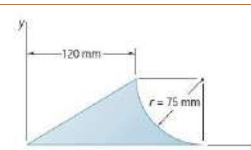
Locate the centroid of the Plane area as shown.



Ans.: $\bar{X}$= -10 mm, $\bar{Y}$= 87.5 mm

Problem 6.2 (Beer Johnston_10th edition_P5.9)

Locate the centroid of the Plane area as shown.

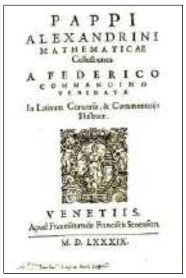


Ans.: $\bar{X}$= 92 mm, $\bar{Y}$= 23.3 mm

Application of Center of Gravity
Area and Volume Determination

- **Theorems of Pappus-Guldinus:**

Two related theorems dealing with the surface areas and volumes of surfaces and solids **of revolution.**



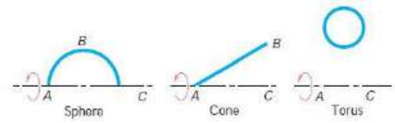
The book of Pappus Alexandrini (290-350)

Mathematician **Paul Galdin** (1577-1663)

Application of Center of Gravity
Pappus-Guldinas Theorem

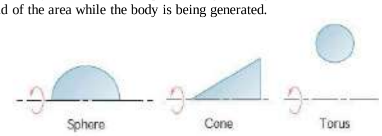
•**THEOREM I**

The area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid of the curve while the surface is being generated.



•**THEOREM II**

The volume of a body of revolution is equal to the generating area times the distance traveled by the centroid of the area while the body is being generated.



Pappus-Guldinas Theorem

•**PROOF I:**

Consider an element dL of the line L, which is revolved about the x axis. The area dA generated by the element dL is,

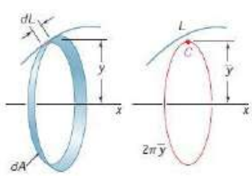
$$dA = 2\pi y\, dL$$

Thus, the entire area generated by L is

$$A = \int 2\pi y\, dL = 2\pi \int y\, dL$$

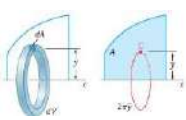
$$\mathbf{A = 2\pi \bar{y} L}$$

where $2\pi y$ is the distance traveled by the centroid of L.



PROOF II:

Similar to Proof I.

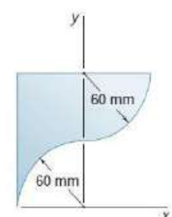


$$\mathbf{V = 2\pi \bar{y} A}$$

<u>Question:</u> Prove the two theorems of Pappus-Guldinus.

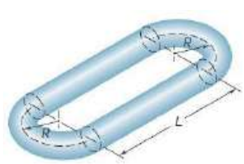
Problem 6.3 (Beer Johnston_10th edition_P5.54)

Determine the volume and the surface area of the solid obtained by rotating the area of about (a) the line x = -60 mm, (b) the line y = 120 mm.



Ans.: (a) $V = 2.26\times10^6$ mm^3, $A = 116.3\times10^3$ mm^2
(b) $V = 1.471\times10^6$ mm^3, $A = 116.3\times10^3$ mm^2

Problem 6.4 (Beer Johnston_10th edition_P5.55)



Determine the volume and the surface area of the solid.

Ans.: V = 3470 mm^3, A = 2320 mm^2

End of Portion 6

References

➢**Vector Mechanics for Engineers: Statics and Dynamics**
Ferdinand Beer, Jr., E. Russell Johnston, David Mazurek, Phillip Cornwell.

➢ **Engineering Mechanics: Statics and Dynamics**
R.C. Hibbeler